VAGUENESS IN DEGREE CONSTRUCTIONS

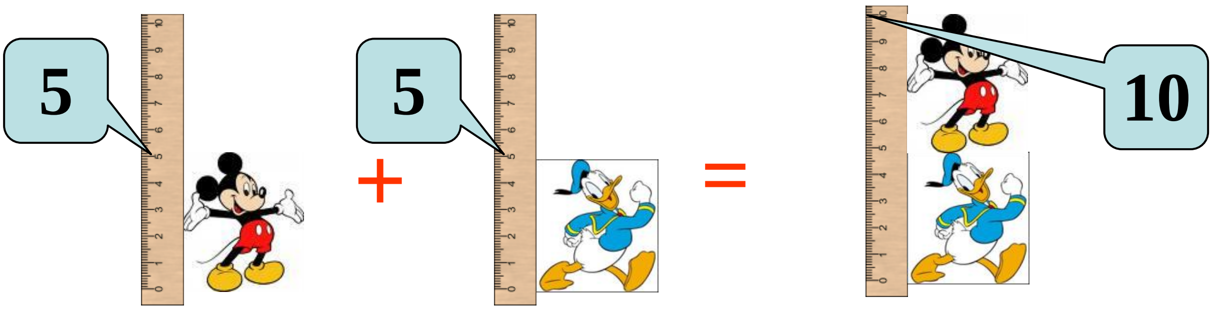
1 THE NATURE OF THE DEGREES IN DIFFERENT GRADABLE PREDICATES

Since Russell (1905), semanticists often characterize gradable predicates as mapping entities to real numbers $r \in \Re$ (Kennedy 1999). The mapping is additive wrt a dimension (Klein 1991).

For example, the degree function of *long*, f_{long}, is 'additive wrt length' in the sense that it adequately represents the fact that the length of the concatenation (placing end to end) of two entities ($d_1 \oplus_{length} d_2$) equals the sum of lengths of the two separate entities (d_1 and d_2):

(1) a. Differences between degrees adequately represent differences between "quantities of length" in entities: $f_{long}(d_1 \oplus_{length} d_2) = f_{long}(d_1) + f_{long}(d_2)$

b. Ratios between degrees adequately represent ratios between "quantities of length" in entities: $f_{long}(d_1) = f_{long}(d_2)$ iff $f_{long}(d_1 \oplus_{length} d_2) = 2 \times f_{long}(d_1)$



This approach provides straightforward semantic accounts of numerical degree predicates (like *2 meters tall*), ratio predicates (like *twice as happy as Sam*), and difference predicates (like *2 meters shorter than Sam*). Yet many object to this approach for several reasons.

1. The problem is mainly that numerical degree modifiers are often infelicitous (#*two meters short*; #*two degrees warm/ happy/ beautiful*). This renders the numerical approach unintuitive.

2. Moreover, there is much indeterminacy concerning the (presumed) mapping of entities to numbers. Given the set of real numbers between 0 and 1, why would one have a degree 0.25 rather than say 0.242 in *happy*? (Kamp and Partee 1995)

3. There is also much indeterminacy regarding the number set itself, e.g., which numbers form the degrees of *happy*?

4. Moltmann (2006) maintains that this indeterminacy creates a meaning intention problem. How do speakers know the meaning of their utterances when they use expressions which (presumably) denote degrees, like (1):

(2) *Dan is as happy as Sam is; Bill isn't that happy*?

Moltmann (2006) concludes that only the few predicates that license numerical degree modifiers map entities to numbers.

2 MY PROPOSAL

Contrary to Moltmann (2006), I propose that any gradable predicate (including *happy*) maps entities to numbers, but in no predicate (including *tall*!) the degree function is fully specified, and this explains the distribution of unit names, numerical degree modifiers, and ratio-modifiers. Let me explain these claims in more details.

Let the set W_c consist of worlds that given the knowledge in some *actual context* c (the common knowledge of some community of speakers out of the blue) may still be the actual world (Stalnaker 1975).

2.1 A New analysis of unit names

We cannot count directly quantities of the 'stuff' denoted by mass nouns (*water, height, heat, happiness*). These quantities have no known values (like 1,2,3,..) Thus, objects d with a non-zero quantity of height (say, the meter) should be mapped to different numerals in different worlds

(3) $\exists w_1, w_2 \in W_c$: $f_{tall,w1}(d) \neq f_{tall,w2}(d)$.

Still, meter rulers tell us the ratios between entities' heights, and in any w, $f_{tall,w}$ represents these ratios.

(4) In every $w \in W_c$, entities with n times d's height are mapped to the numeral $n \times f_{tall,w}(d)$.

All *tall*'s functions in W_c, then, yield the same ratios between entities' degrees (these ratios are known numbers). Hence, I propose the following analysis of unit names.

(5) Unit names like *meter* have two interpretations:
1. The first one, *meter*[1], is a one-place predicate that denotes (a characteristic function of) a set of equally tall entities, the *meter unit objects* (those entities, whose height we call ‘1 meter’).
2. The second 'numerical-degree modifier' interpretation, *meter*[2], is a two place predicate – a relation between a number n and an object x, s.t. x is n times as tall as a meter unit-object.

(6) $[[\text{Dan is two-meters tall}]]_c = 1$ iff $\forall w \in W_c$: $f_{tall,w}([[\text{Dan}]]_c) = 2 \times r_{m,w}$.

Therefore, it is not the case that Dan is 2 meters tall iff f_{tall} maps Dan to 2. The value to which f_{tall} maps Dan is unknown

(7) $\neg\exists n \in \Re$: $\forall w \in W_c$, $f_{tall,w}([[\text{Dan}]]_w) = n$.

We feel that we have knowledge about entities' degrees in *tall* only because **the following two preconditions hold**:

(i) The ratios between entities' degrees are known numbers ($\forall d_1,d_2 \in D$, $\exists n \in \Re$: $\forall w \in W$, $f_{tall,w1}(d_1) / f_{tall,w1}(d_2) = n$), and

(ii) There is an agreed upon set of unit-objects (e.g., the meters) that serve as a reference point, such that any entity d is mapped to a known number, *representing the ratio between d's degree and the unit objects' degree* in *tall*.

2.2 Violations of (ii): **Lack of agreed-upon unit-objects**

My proposal predicts that the lack of conventional unit objects will result in vagueness concerning the mapping of entities to numbers.

Consider, for instance, *happy*. Emotions are internal states. It is hard to come up with conventions as to which emotional extent should be mapped to degree 1, 2, 3, etc. Even if one speaker treats a certain internal state as a unit object, no other speaker has access to this state. So no object d can be such that it would be *agreed upon by all the community of speakers* that d is a unit object.

Similarly, while *weight* can be measured by kilograms, the internal states of speakers when they lift objects (their feeling of the objects being *heavy*, *light*, etc.) cannot be measured by conventionally established unit names.

If a language maps a predicate to the latter type of degrees *the predicate will not license unit names and numerical degree modifiers*. But this does not show that predicates do not map entities to numbers (contrary to Moltmann's 2006 view). In fact, when no unit name is explicitly mentioned, it is rather meaningless to say that something is tall to degree 456 (456 what? Kilometers? Meters? Inches?) In adjectives like *happy* this is always the situation.

This proposal is superior to non-numerical theories (cf. Moltmann 2006) because it accounts for the compatibility of *happy* with ratio and difference modifiers.

For example, *Dan is twice as happy as Sam* is a claim about ratios between *happiness* degrees.

(8) [[*Dan is twice as happy as Sam*]]$_c$ = 1 iff
$\forall w \in W_c$: $f_{happy,w}([[Dan]]_w) = 2 \times f_{happy,w}([[Sam]]_w)$.

In addition, we can give a unified analysis to difference predicates with and without numerical degree modifiers

(9) a. [[Dan is *2 meters taller* than Sam]]$_c$ = 1 iff
$\forall w \in W_c$: $f_{tall,w}(Dan) - f_{tall,w}(Sam) = 2 \times r_{m,w}$;
b. [[Dan is *happier* than Sam]]$_c$ = 1 iff
$\forall w \in W_c$, $\exists r \in \Re$, $r>0$: $f_{happy,w}(Dan) - f_{happy,w}(Sam) = r$.

Finally, no meaning intention problem arises, as speakers do not need to know the entities' degrees, only the ordering or ratios between their potential degrees.

2.3 Violations of (i): **Lack of knowledge about ratios between degrees**

While we may feel acknowledged of the ratios between, say, our degrees of happiness in separate occasions, we can hardly ever feel acknowledged of the ratios between degrees of entities in predicates like *short*. This is illustrated by the fact that ratio modifiers are less acceptable with *short* than with *tall* or with *long* (as in (10)), and less often used with *short* (as Google search-results show).

(10) *Dan is twice as tall as Sam* vs. #*Dan is twice as short as Sam*),

In accordance, the present analysis predicts that, in the lack of knowledge concerning ratios between degrees, numerical degree predicates will not be licensed (as in **two meters short*).

Still, numerical degree predicates *are* fine in the comparative (as in *two meters shorter*). In actual contexts, we can positively say that Dan's degree in *short* is n meters bigger than Sam's iff Sam's degree in *tall* is n meters bigger than Dan's.

In a different paper (Salt 18), I make the case for the claim that any function that linearly reverses and linearly transforms the degrees of f_{tall} can predict these facts. In other words, I propose that for any $w \in W_c$ there is a constant $Tran_{short,w} \in \Re$, such that $f_{short,w}$ assigns any d the degree ($Tran_{short,w} - f_{tall,w}(d)$) (so Dan is taller iff Sam is shorter).

(11) $\forall w \in W_c, \exists Tran \in \Re$, s.t. $\forall d \in D, f_{short,w}(d) = Tran - f_{tall,w}(d)$

Now, intuitively, in every $w \in W_c$, $f_{tall,w}$ maps entities with no height (abstract entities; surfaces) to 0 (it's additive), but $f_{short,w}$ doesn't (according to some semantic theories the degree of such entities in *short* approximates infinity; Kennedy 1999). I.e. $f_{short,w}$ transforms height quantities by a constant, $Tran_{short,w}$. The transformation value, $Tran_{short}$, is unknown.

(12) $\neg\exists n \in \Re: \quad \forall w \in W_c, Tran_{short,w} = n$.

But when $Tran_{short,w} \neq 0$, $f_{short,w}$ isn't additive (doesn't represent ratios between entities' heights).

If, for instance, $f_{tall,w}(d_1)=f_{tall,w}(d_2)=5$, then by additivity $f_{tall,w}(d_1 \oplus d_2)=10$. But, say, a function f_{1-f} that maps each d to ($1 - f_{tall,w}(d)$) is s.t. ($f_{1\text{-}f}(d_1)= f_{1\text{-}f}(d_2)=-4$) and ($f_{1-f}(d_1 \oplus_{height} d_2)= 1 - f_{tall,w}(d_1 \oplus_{height} d_2) = -9 \neq (2 \times -4)$). The ratio between the degrees of $d_1 \oplus_{height} d_2$ and d_1 is 9/4, while the ratio between their heights is 8/4.

Since f_{short} fails to represent height ratios:

- Ratio modifiers are less acceptable with *short* than with *tall* (*#Dan is twice as short as Sam*), as Google search-results show.
- Numerical degree predicates can't be used with *short* (as in **two meters short*).
  If, for instance, in c, *tall* maps some d to 2 meters, *short* maps d to $Tran_{short}$ – 2 meters.

(13) ($\forall w \in W_c, f_{tall,w}(d) = 2 \times r_{m,w}$) iff ($\forall w \in W_c, f_{short,w}(d) = Tran_{short,w} - 2 \times r_{m,w}$)

So in the lack of knowledge about $Tran_{short}$ (given that it varies across worlds in W_c), we can't say which entities are *2 meters short* in c.

(14) $\neg\exists d: \quad \forall w \in W_c, f_{short,w}(d) = 2 \times r_{m,w}$.

- However, in computing degree-differences, the transformation values cancel one another:

(15) a. $\forall w \in W_c$, d_2 is 2 meters taller than d_1
($f_{tall,w}$ maps d_2 to some $n \in \Re$ and d_1 to $n - 2 \times r_{m,w}$) iff
b. $\forall w \in W_c$, d_1 is 2 meters shorter
($f_{short,w}$ maps d_2 to $Tran_{short,w} - n$ and d_1 to $Tran_{short,w} - (n - 2 \times r_{m,w})$;
The degree difference is still $2 \times r_{m,w}$)

Thus, we can felicitously say that some entity-pairs stand in the relation '*two meters shorter*'.

Last but not least, my proposal is superior to other accounts of the licensing of numerical degree predicates (von Stechow 1984; Kennedy 1999), because it captures facts pertaining to positive predicates, like *warm*. Positive predicates may have transformation values, too, which (among other things) render, *#2 degrees warm*, but not *2 degrees warmer*, infelicitous.

3 VAGUENESS OF A DIFFERENT SOURCE

We may not know whether sentences like (16) are true or not.

(16) *Dan is (two inches / three times) taller than Sam*.

If the referent of *Dan* in w_1 is 1.87 meters tall, and the referent of *Dan* in w_2 is 1.86 meters tall, I say (following Lewis 1986) that the name *Dan* refers to two different individuals in these two worlds.

However, if in w_1 and w_2 the referent of *Dan* is 1.87 meters tall, and identical in all the other property values, even if 1.87 counts as 'tall' in w_1 but not in w_2, I still say (unlike Lewis 1986) that the name *Dan* denotes the same individual in these two worlds (it is only our interpretation of the word *tall* that has changed).

I do take individuals to be real entities, identified with their 'real' properties. So it is invariably determined for each two individuals in D what their heights are. However, when we use proper names, we do not know exactly which individuals in D they refer to (since we do not know all of their property values).

4 SEMANTIC RELATIVITY

Future research should establish whether judgments about *That was fun*/ *tasty* that form a basis for a relativist semantics (Lasersohn 2005; Stephenson 2007) can be accounted for without a move to full-blown relativity (I thank the referee for bringing this up). I propose that such judgments involve measuring extents of internal states. This directly predicts that people who associate such predicates with measures of their own inner states may vary in judgment (which may seem contradictory if we expect our inner extents to be similar).

PREDICATE TYPES

Table 1: Predicate types		
Ratio	Interval	Ordinal
Knowledge about ratios: *Dan is twice as tall as Sam* *Dan is twice as happy as Sam*	**No** knowledge about ratios: *#Dan is twice as short as Sam* *# Dan is twice as unhappy …*	**No** knowledge about ratios: *#Tweety is twice as a bird as Tan* *#…twice as "bald and tall" as Tan* (where *twice* takes scope over *and*)
Knowledge about intervals: *Dan is 2 inches taller than Sam*	Knowledge about intervals: *Dan is 2 inches shorter than …*	**No** knowledge about intervals: *#Tweety is more a bird than Tan*
Knowledge about ordering: *Dan's degree (the extent it satisfies the property) 'tall' is bigger than Sam's*	Knowledge about ordering: *Dan's degree in (the extent it satisfies the property) 'short' is bigger than Sam's*	Knowledge about ordering: *Tweety's degree in (the extent it satisfies the property) 'bird' is bigger than Tan's*
- Many measures of physical quantities, such as mass, length, and energy - Measures of age - Length of residence in a place.	- Negated predicates - Many negative predicates (*short*) - Positive predicates where intuitions about the zero point are blurred (*warm*)	- Nominal concepts (nouns and noun phrases) - Conjunctions, Disjunctions - Conjunctive or disjunctive adjectives (*healthy wrt blood pressure, pulse and body fever*)
Tall, long, wide, big, hot, weighs, happy, $\text{cold}_{\text{mental}}$	$\text{cold}_{\text{temperature}}$, warm, heavy, short, small, light	Bird, apple, tall person, fat or bald, fat and happy

5 CELSIUS

The interpretation of *Celsius* is not generated by the general 'linguistic' rule for the interpretation of unit names proposed above. It is a 'derived' unit. For any n, entities "n Kelvin hot" are "n – 273 Celsius hot":

Kelvin degree	Celsius degrees
0	0 – 273 = – 273
1	1 – 273 = – 272
273	273 – 273 = 0
274	274 – 273 = 1
275	275 – 273 = 2

So things are *1 degree Celsius* iff they are *274 degrees Kelvin*. But a box is *1 degree Celsius more* than a shelf iff the box is *1 degree Kelvin more* than the shelf, not *274 degrees Kelvin* more.

The numbers that *Celsius* assigns to entities are not additive. They do not adequately represent the fact that the heat in two rods together equals the sum of heats in the two separate rods.

For example, if rod d_1 and d_2 each contains heat of 2 Kelvin degrees ($2\times r_{Kelvin}$), each falls under "2 – 273 Celsius hot", and the heat contained in both of them together, the heat in 4 Kelvin unit objects ($4\times r_{Kelvin}$), falls under "(4 – 273) Celsius hot". But $(2 - 273) + (2 - 273) = (4 - 546) \neq (4 - 273)$. Thus, $Celsius^2$ does not assign $d_1 \oplus d_2$ the sum of the numbers it assigns to d_1 and d_2. The heat in any entity which is "2 Celsius hot" is not twice the heat in an entity which is "1 Celsius hot".

In fact, the handbooks are full with explanations as to why it is senseless to say that "*4 Celsius is twice as hot as 2 Celsius*". But linguistically, speakers analyze *Celsius* as a unit name, not a predicate with Kelvin units, so despite these explanations, they cannot help feeling that this sentence is just fine (just like the sentence "*4 meters is twice as long as 2 meters*"). This further supports my proposal that when speakers use a unit name, they presuppose that the transformation value of the predicate it is a unit of is set to zero. These speakers analyze the statement *The box is 2 Celsius hotter than the shelf* as if it means: " $f_{hot}([[\text{the box}]]) = f_{hot}([[\text{the shelf}]]) + 2\times r_{Celsius}$", which is, of course, wrong (the difference is $2\times r_{Kelvin}$).

$Celsius_{hot}$ seems to best be analysed as a 'normal' predicate, not a unit name. One's degree in $Celsius_{hot}$ is one's degree in Kelvin (the ratio between one's heat and the heat in a Kelvin unit object) – minus 273.

(17) For any d, $f_{Celsius\text{-}hot}(d) = f_{hot}(d)/r_{Kelvin} - 273$

(18) [[*The box is 2 degrees Celsius hotter than the shelf*]$_c$ = 1 iff

$f_{Celsius\text{-}hot}([[\text{the box}]]^{+}{}_{c}) = f_{Celsius\text{-}hot}([[\text{the shelf}]]^{+}{}_{c}) + 2$

iff $\exists d_k \in [[\text{Kelvin}^1]]_c$, $f_{hot}([[\text{the box}]]_c)/ f_{hot}(d_k) - 273 = f_{hot}([[\text{the shelf}]]_c)/ f_{hot}(d_k) - 273 + 2$

iff $f_{hot}([[\text{the box}]]_c) = f_{hot}([[\text{the shelf}]]_c) + 2\times r_{Kelvin}$

The ratio of heat in the box and in a Kelvin unit-object equals the ratio of heat in the shelf and in a Kelvin unit-object plus two (so the box is 2 degrees Kelvin hotter).